Doubly Nonstationary Blind Source Separation

Adrien Meynard

Aix Marseille Univ, CNRS, Centrale Marseille, I2M, Marseille, France

Aix*Marseille université — Initiative d'excellence

Context:
- ▶ Blind source separation problem
- ▶ Nonstationary sources
- ▶ Time varying mixing matrix

Goal:
- ▶ Estimate sources from the observations of their linear mixture ⇒ BSS
- ▶ Estimate deformation functions (time warping) which characterize the nonstationarity of the sources } JEFAS
- ▶ Estimate spectra of the underlying stationary sources } JEFAS

I. Model

A class of nonstationary signals

▶ x is a realization of a stationary random process X with power spectrum $\mathcal{S}_X$. Acting on x with a time warping operator yields a nonstationary signal y given by:

$$y(t) = \sqrt{\gamma'(t)}\, x(\gamma(t)) \ .$$

▶ Wavelet transform:

$$\mathcal{W}_x(s,\tau) = \int_{\mathbb{R}} x(t) q^{-\frac{s}{2}} \overline{\psi}\left(\frac{t-\tau}{q^s}\right) dt \quad \text{with} \quad q > 1 \ .$$

⇒ The wavelet transforms of the nonstationary signal y and the underlying stationary signal x are approximately related as follows:

$$\mathcal{W}_y(s,\tau) \approx \mathcal{W}_x(s + \log_q(\gamma'(\tau)), \gamma(\tau)) \ .$$

It can be shown [1] that the approximation error is zero-mean and its variance is controlled by the decay properties of the wavelet ψ, and the variations of γ.

▶ JEFAS [2] is an algorithm which relies on the approximation equation for the joint estimation of the time warping function γ' and the spectrum $\mathcal{S}_X$.

Doubly nonstationary instantaneous mixture

▶ Mixture model:
- ▸ Sources $\mathbf{y}(t) \in \mathbb{R}^N$: nonstationary signals (time warping model).
- ▸ Observations $\mathbf{z}(t) \in \mathbb{R}^N$: instantaneous linear mixtures of the sources.

$$\mathbf{z}(t) = \mathbf{A}(t)\mathbf{y}(t) \ .$$

▶ **Goal:** determine jointly the mixing matrix $\mathbf{A}(t)$, the time warping functions $\gamma_i(t)$, and the spectra $\mathcal{S}_{X_i}$ of the stationary sources for $i \in \{1,\dots,N\}$ from the observations $\mathbf{z}(t)$.

▶ Approximation equation:
- ▸ Assumption: the mixing matrix coefficients are slowly varying.
- ▸ ⇒ At fixed time τ: approximate linear relation between the wavelet transforms of the sources and the observations:

$$\mathbf{w}_{\mathbf{z},\tau} \approx \mathbf{A}(\tau)\mathbf{w}_{\mathbf{y},\tau} \ .$$

where $\mathbf{w}_{\mathbf{z},\tau} = \left(\mathbf{w}_{z_1,\tau}^T \cdots \mathbf{w}_{z_N,\tau}^T\right)^T$ with $\mathbf{w}_{z_i,\tau} = \mathcal{W}_{z_i}(\mathbf{s},\tau)$. The same notation is used for the wavelet transform of the sources $\mathbf{w}_{\mathbf{y},\tau}$.

II. Estimation procedure and algorithm

For each τ, the parameters to estimate are:
- ▸ The unmixing matrix: $\mathbf{B}_\tau = \mathbf{A}(\tau)^{-1}$
- ▸ The time warping parameters: $\boldsymbol{\theta}_\tau = (\theta_{1,\tau} \cdots \theta_{N,\tau})^T$ where $\theta_{i,\tau} = \log_q(\gamma_i'(\tau))$

Probabilistic setting

▶ Assume the stationary sources X_i are Gaussian. Then, we derive the law of the wavelet transforms of the sources $\mathbf{w}_{\mathbf{y},\tau}$ from the approximation equations. And, one can write the approximated likelihood on $\mathbf{w}_{\mathbf{z},\tau}$ given by

$$\ell_\tau(\mathbf{B}_\tau, \boldsymbol{\theta}_\tau) = \log(p_{\mathbf{w}_{\mathbf{z},\tau}|(\mathbf{B}_\tau,\boldsymbol{\theta}_\tau)}(\mathbf{w}_{\mathbf{z},\tau}; \mathbf{B}_\tau, \boldsymbol{\theta}_\tau))$$

▶ Besides, a regularity prior is given to $\mathbf{B}_\tau$ in order to take into account the smoothness assumption on the mixing matrix.

t y Estimation algorithm: JEFAS-BSS

▶ Initialization: Evaluate $\tilde{\mathbf{B}}_\tau^{(0)}$ ⇒ Estimate the sources via $\tilde{\mathbf{y}}^{(0)}(\tau) = \tilde{\mathbf{B}}_\tau^{(0)}\mathbf{z}(\tau)$.

▶ The following two steps are computed alternatively until convergence:

1. For each source, estimate parameters $\tilde{\theta}_{i,\tau}^{(k)}$, $\forall\tau$ and spectrum $\tilde{\mathcal{S}}_{X_i}^{(k)}$ applying JEFAS to $\tilde{y}_i^{(k-1)}$.
2. For each τ, solve the MAP problem replacing $\boldsymbol{\theta}_\tau$ with its current estimation:

$$\tilde{\mathbf{B}}_\tau^{(k)} = \arg\max_{\mathbf{B}_\tau} \ \ell_\tau(\mathbf{B}_\tau, \tilde{\boldsymbol{\theta}}_\tau^{(k)}) \quad \text{s.t.} \quad \|\mathbf{B}_\tau - \tilde{\mathbf{B}}_{\tau-\Delta_\tau}^{(k)}\|_\infty \leq \epsilon_B \Delta_\tau .$$

⇒ Estimate the sources via $\tilde{\mathbf{y}}^{(k)}(\tau) = \tilde{\mathbf{B}}_\tau^{(k)}\mathbf{z}(\tau)$

III. Results

The sharp wavelet $\psi_\sharp$ (with infinitely many vanishing moments) is used, supported in the positive Fourier domain and defined by

$$\hat{\psi}_\sharp(\omega) = \epsilon^{\frac{\delta(\omega,\omega_0)}{\delta(\omega_1,\omega_0)}} \ , \ \omega > 0 \text{ where } \delta(a,b) = \frac{1}{2}\left(\frac{a}{b} + \frac{b}{a}\right) - 1 \ .$$

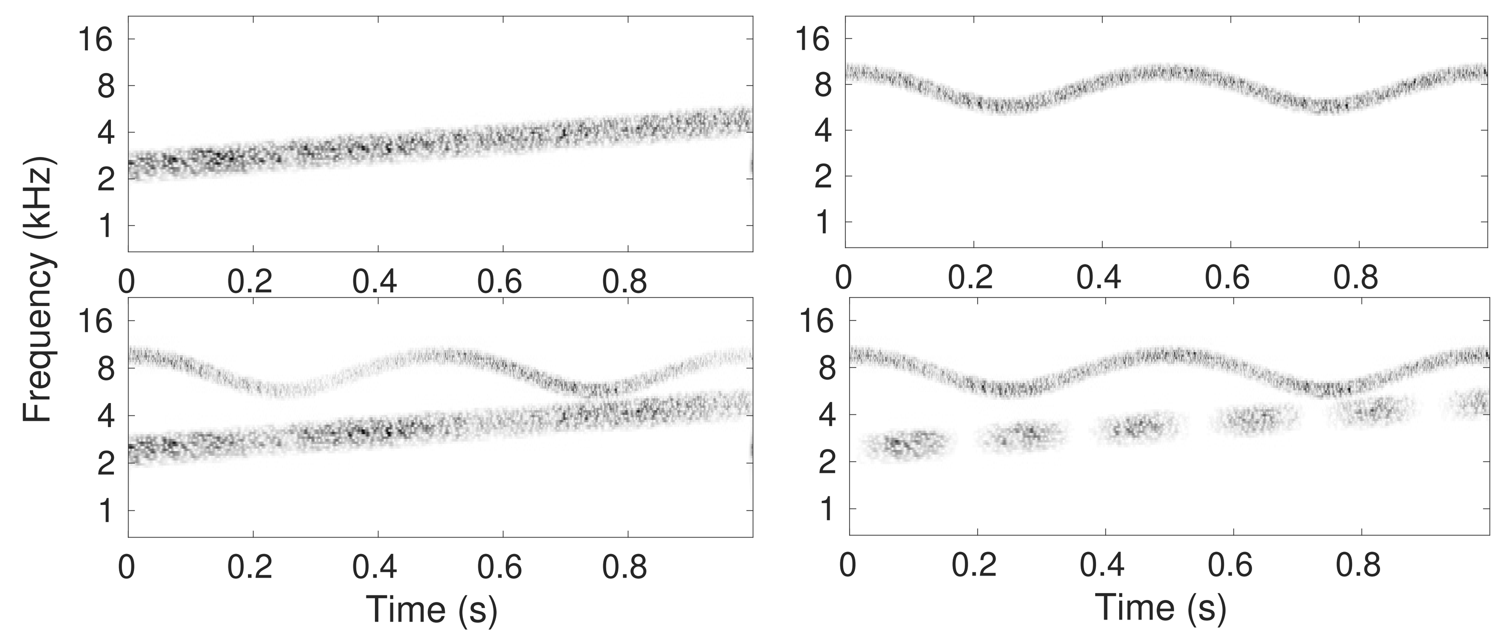
Here ω_0 is the mode of $\hat{\psi}$, ω_1 is chosen so that $\hat{\psi}_\sharp(\omega_1) = \epsilon$.

Synthetic example

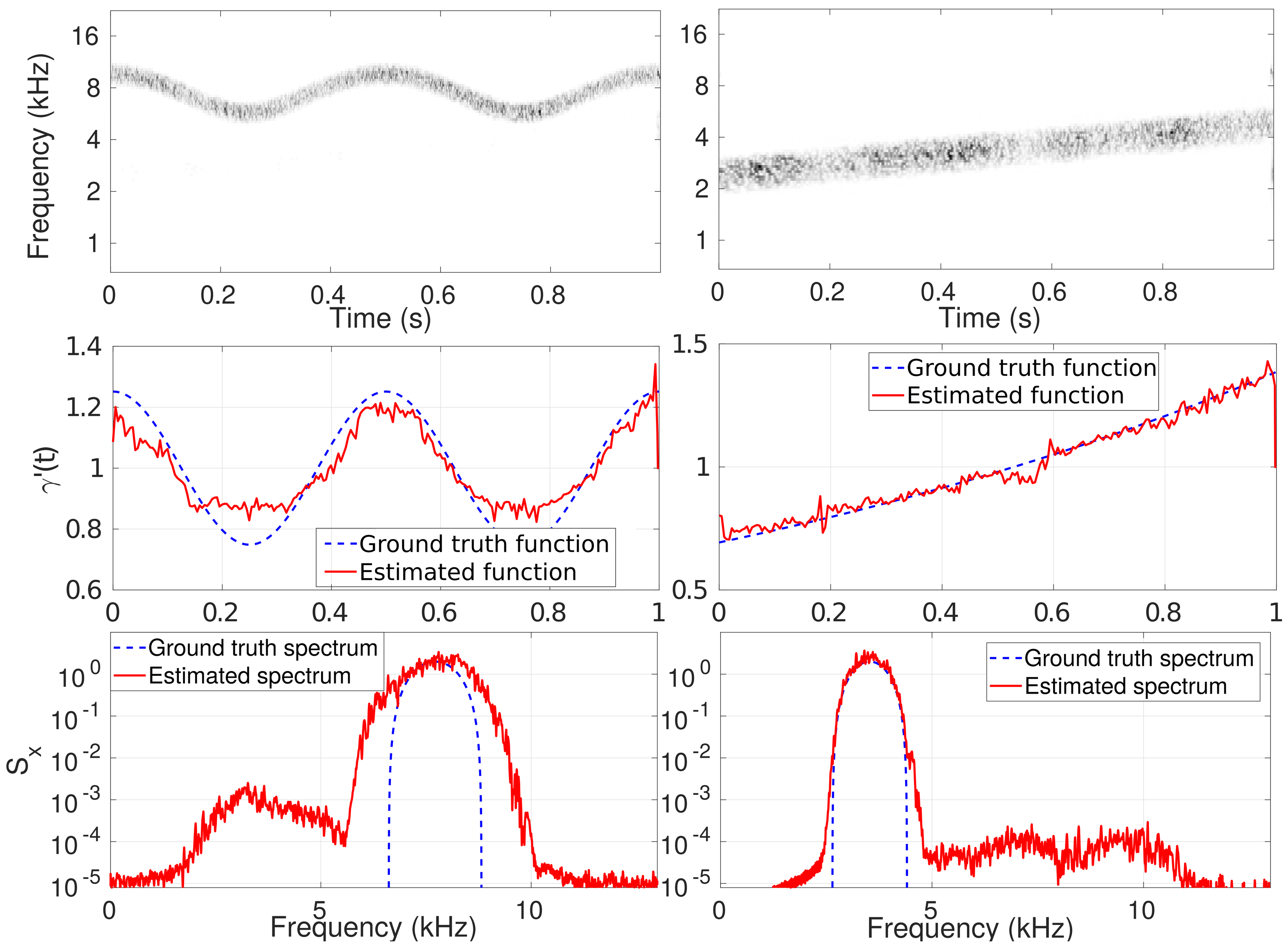
- ▸ $N = 2$
- ▸ Both nonstationary sources (time warping model) are Gaussian. Their underlying power spectra are two non-overlapping Hann windows.
- ▸ The time-varying mixing matrix coefficients are sinusoidally varying over time:

$$\mathbf{A}(t) = \begin{pmatrix} 1 + 0.3\cos(5\pi\frac{t}{T}) & 0.75 + 0.4\cos(3\pi\frac{t}{T}) \\ -0.5 + 0.5\cos(11\pi\frac{t}{T}) & 1 + 0.1\cos(8\pi\frac{t}{T}) \end{pmatrix} \ .$$

⇒ The wavelet transforms of both sources and observations are displayed below.



▶ JEFAS-BSS converges in 8 iterations. The estimated sources are displayed below, together with the corresponding estimated time warping functions and spectra.



▶ We compare the performances of JEFAS-BSS with reference BSS algorithms adapted to stationary sources (SOBI [3]) or constant mixing matrix (QTF-BSS [4]).

Algorithm	SIR (dB)		Amari index (dB)	
	Mean	SD	Mean	SD
SOBI	24.12	0.34	−7.13	0.01
p-SOBI	5.27	2.91	−9.38	0.03
QTF-BSS	24.73	0.63	−7.15	0.02
JEFAS-BSS	35.61	2.61	−25.80	0.39

Table: Comparison of the Signal to Interference Ratio and the averaged Amari index for four BSS algorithms.

References

[1] Harold Omer and Bruno Torrésani. Time-frequency and time-scale analysis of deformed stationary processes, with application to non-stationary sound modeling. *Applied and Computational Harmonic Analysis*, 43(1):1 – 22, 2017.

[2] Adrien Meynard and Bruno Torrésani. Spectral Analysis for Nonstationary Audio. *IEEE/ACM Transactions on Audio, Speech and Language Processing*, 26(12):2371 – 2380, December 2018.

[3] Adel Belouchrani, Karim Abed-Meraim, Jean-François Cardoso, and Eric Moulines. A blind source separation technique using second-order statistics. *IEEE Transactions on Signal Processing*, 45(2):434–444, Feb 1997.

[4] Nadège Thirion-Moreau and Moeness G. Amin. Chapter 11 - quadratic time-frequency domain methods. In P. Comon and C. Jutten, editors, *Handbook of Blind Source Separation*, pages 421 – 466. Academic Press, Oxford, 2010.