Reduction of boundary effects in real-time time-frequency analysis

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Joint work with Hau-Tieng Wu

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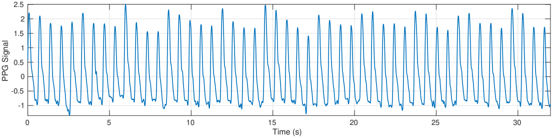
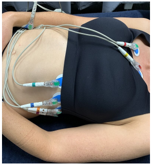
Outline

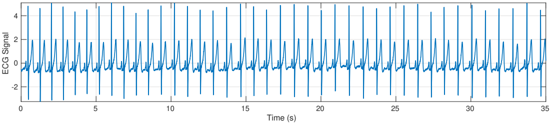
Biomedical signals: Cardiac signals

- Blood volume changes $\leftrightarrow$ Photoplethysmogram (PPG)

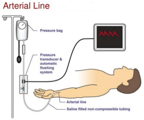


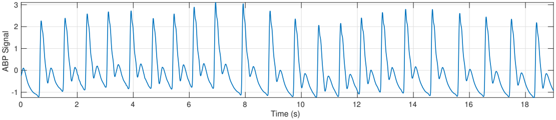
- Electrical activity of the heart $\leftrightarrow$ Electrocardiogram (ECG)





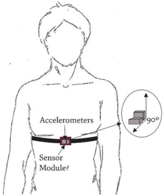
- Arterial blood pressure (ABP)

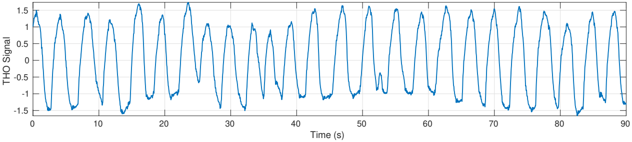




Biomedical signals: Respiratory signals

- Thoracic movement recorded by a piezoelectric sensor (THO)



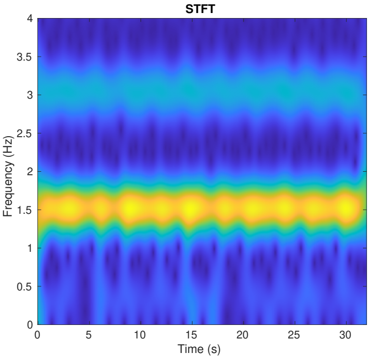


Time-frequency analysis

Goal: Visualize the evolution of the instantaneous frequencies forming the signal.

$\Rightarrow$ Time-frequency representations.

- Short-Time Fourier Transform (STFT);
- Synchrosqueezing Transform (SST);
- Reassignment (RS);
- ...



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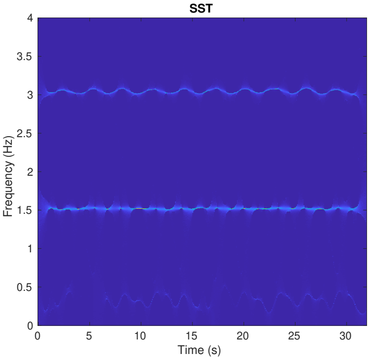

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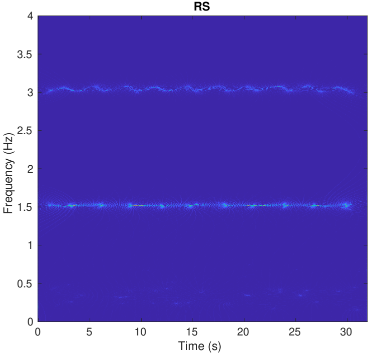

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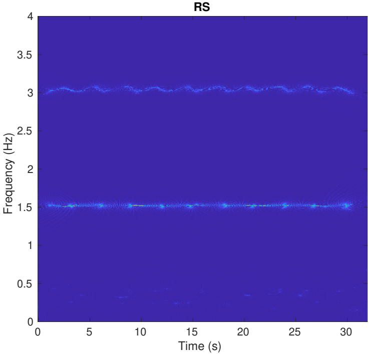
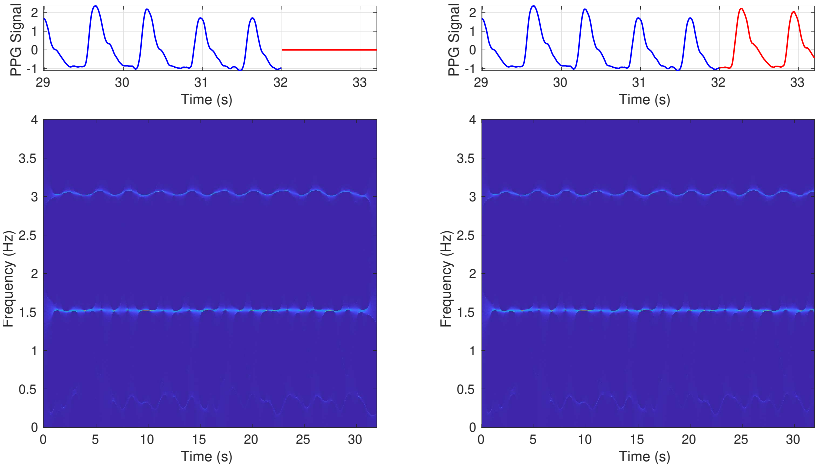


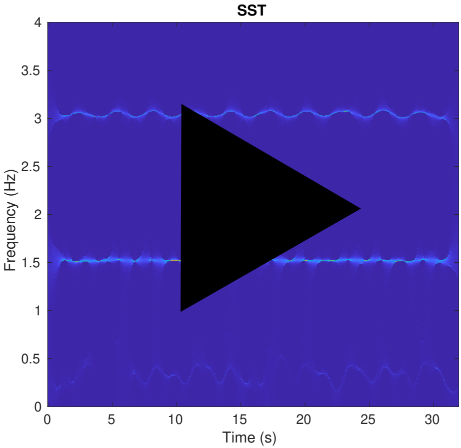
Boundary effects

- Time-frequency analysis is based on windowing techniques.
- Time-frequency representations are thus sensitive to boundaries.
- The accuracy of these representation is deteriorated near boundaries.

$\Longrightarrow$ **Boundary effects**



Real-time time-frequency analysis



Goal: Being able to display real-time boundary-free time-frequency representations.

1 Introduction: boundary effects

2 A boundary-effect reduction algorithm

3 Numerical results

4 Conclusion

Strategy

Two-step strategy:

1. Forecast a sufficiently long extension of the signal to push signal's boundaries;
2. Perform a time-frequency analysis on the extended signal, until the current time.

$\Rightarrow$ How can we forecast the biomedical signals that interest us?

Extension step

Idea: Take advantage of the shape of the signals we consider to establish a **fast** and **accurate** forecasting method.

Adaptive Harmonic Model

- f_s is the sampling frequency;
- $\mathbf{x} \in \mathbb{R}^N$ is the observed signal;
- $\mathbf{w} \in \mathbb{R}^N$ is a Gaussian white noise;
- $\mathbf{z} \in \mathbb{R}^N$ is an oscillatory deterministic signal such that

$$\mathbf{z}[n] = \sum_{j=1}^{J} a_j\left(\frac{n}{f_s}\right)\cos\left(2\pi\phi_j\left(\frac{n}{f_s}\right)\right),$$

where $a_j\left(\frac{n}{f_s}\right)$ and $\phi_j'\left(\frac{n}{f_s}\right)$ describe how large and fast the signal oscillates at time $\frac{n}{f_s}$.

Then, we assume that

$$\mathbf{x} = \mathbf{z} + \sigma\mathbf{w}\,.$$

Extension step

To determine a prediction of the signal of length L, we estimate the dynamical behavior of the oscillating signal from its previous values.

1. We extract a set $\{\mathbf{x}_1, \ldots, \mathbf{x}_K\}$ of K subsignals of length $M > L$ from $\mathbf{x}$.
2. Establishing a dynamical model consists in determining the relationship that binds $\mathbf{x}_{k+1}$ to $\mathbf{x}_k$, that is finding a function f so that

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k), \quad \forall k \in \{1, \ldots, K-1\} .$$

Extension step

Here, we consider here a naive dynamical model, assuming that we have

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k \,, \quad \forall k \in \{1, \ldots, K-1\} \,,$$

where $\mathbf{A} \in \mathbb{R}^{M \times M}$.

- Classical strategy in the study of dynamical systems: *linearization of a nonlinear system*.
- Fast and simple estimation of the parameters characterizing the dynamical model. Indeed:

$$\underbrace{\tilde{\mathbf{A}} = \mathbf{Y}\mathbf{X}^T(\mathbf{X}\mathbf{X}^T)^{-1}}_{\text{Least square estimator}} \,.$$

  where $\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 & \cdots & \mathbf{x}_{K-1} \end{pmatrix}$ and $\mathbf{Y} = \begin{pmatrix} \mathbf{x}_2 & \cdots & \mathbf{x}_K \end{pmatrix}$.
- The forecasting of $\mathbf{x}_{k+\ell}$ is obtained by recursively applying the linear relation:

$$\tilde{\mathbf{x}}_{K+\ell} = \underbrace{\tilde{\mathbf{A}}\tilde{\mathbf{A}} \cdots \tilde{\mathbf{A}}}_{\ell \text{ times}} \mathbf{x}_K = \tilde{\mathbf{A}}^\ell \mathbf{x}_K \,.$$

$\Longrightarrow$ Fast forecasting strategy. Is it accurate?

Theoretical forecasting error

In the case where $\mathbf{x}$ follows the AHM model, we asymptotically bound the forecasting error ϵ defined by

$$\epsilon[n] = \tilde{\mathbf{x}}[n] - \mathbf{z}[n] \ .$$

In particular, we focus on

1. The bias, such that

$$\boldsymbol{\mu}[n] = \mathbb{E}\{\epsilon[n]\} \ ;$$

2. The variance, given by

$$\gamma[n, n'] = \mathbb{E}\{(\epsilon[n] - \boldsymbol{\mu}[n])\,(\epsilon[n'] - \boldsymbol{\mu}[n'])\} \ .$$

Theoretical forecasting error

Theorem (Asymptotic behavior the the forecasting error)

- *The first-order moment of the forecasting error at time $n \geq N$ satisfies*

$$|\boldsymbol{\mu}[n]| \leq a_0^{(n)}\sigma^2 + \frac{1}{K}\left(\frac{a_1^{(n)}}{\sigma^2} + a_2^{(n)}\right) + o\left(\frac{1}{K}\right)$$

as $K \to \infty$, where $a_0^{(n)}$, $a_1^{(n)}$, and $a_2^{(n)}$ are positive quantities, independent of K and σ.

- *Its second-order moment satisfies:*

$$|\gamma[n, n']| \leq c_0^{(n,n')}\sigma^2 + \frac{1}{K}\left(\frac{c_1^{(n,n')}}{\sigma^2} + c_2^{(n,n')} + c_3^{(n,n')}\sigma^2\right) + o\left(\frac{1}{K}\right)$$

as $K \to \infty$, where $c_0^{(n,n')}$, $c_1^{(n,n')}$, $c_2^{(n,n')}$ and $c_3^{(n,n')}$ are positive quantities, independent of K and σ.

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3 Numerical results
- Performance of the extension scheme
- Performance of the boundary effect reduction

4 Conclusion

Illustration of the theoretical results on a simulated signal

- Evolution of the experimental forecasting variance as a function of the noise variance for three different values the forecasting sample index ℓ.

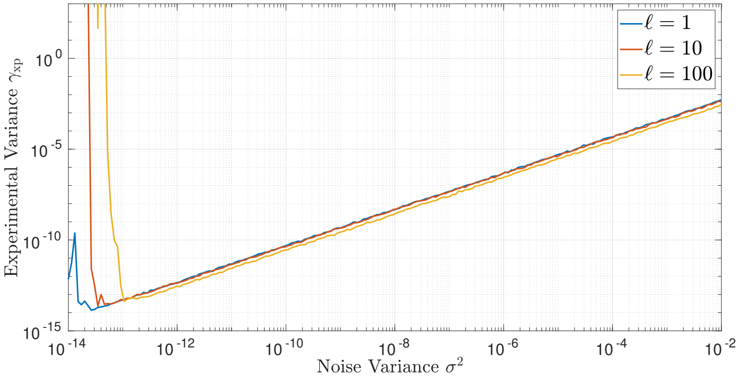
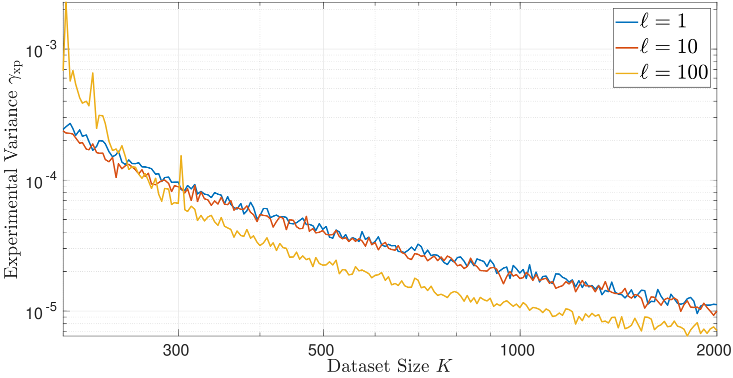


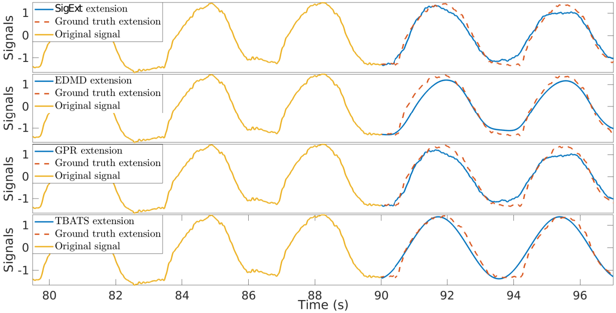
Illustration of the theoretical results on a simulated signal

- Evolution of the experimental forecasting variance in function of the dataset size for three different values the forecasting sample index ℓ.



Applications to Real Physiological Signals

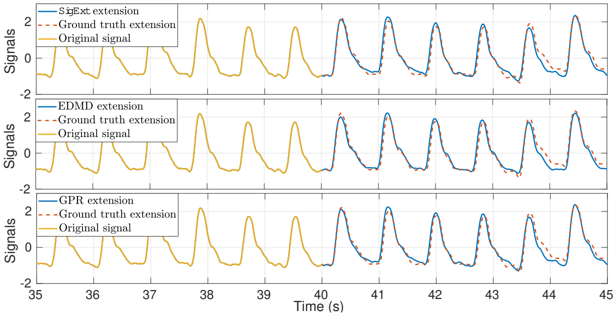
Respiratory signal



Extension method	MSE	Performance index D (mean $\pm$ SD)			
		STFT	SST	RS	ConceFT
SigExt	0.292 $\pm$4.438	0.370 $\pm$0.623	0.408 $\pm$0.436	0.866 $\pm$0.879	0.423 $\pm$0.344
Symmetric	0.044 $\pm$0.111	1.162 $\pm$0.893	1.173 $\pm$0.886	1.022 $\pm$0.281	1.144 $\pm$0.579
EDMD	0.026 $\pm$0.112	0.359 $\pm$0.266	0.422 $\pm$0.282	0.828 $\pm$0.248	0.449 $\pm$0.296
GPR	0.331 $\pm$4.858	0.391 $\pm$0.853	0.411 $\pm$0.406	0.897 $\pm$1.140	0.430 $\pm$0.364

Applications to Real Physiological Signals

PPG signal



Extension method	MSE	Performance index D (mean $\pm$ SD)			
		STFT	SST	ConceFT	RS
SigExt	0.018 $\pm$0.014	0.280 $\pm$0.107	0.309 $\pm$0.112	0.367 $\pm$0.183	0.534 $\pm$0.160
Symmetric	0.037 $\pm$0.007	1.168 $\pm$0.390	1.209 $\pm$0.340	1.310 $\pm$0.140	0.983 $\pm$0.304
EDMD	0.012 $\pm$0.005	0.289 $\pm$0.126	0.319 $\pm$0.134	0.375 $\pm$0.163	0.503 $\pm$0.163
GPR	0.018 $\pm$0.013	0.276 $\pm$0.106	0.303 $\pm$0.110	0.361 $\pm$0.165	0.544 $\pm$0.157

Real-Time Implementation

- f_s: sampling frequency
- t_{forecast}: time to compute the forecasting of L oncoming samples
- t_{SST}: time to compute one column of the synchrosqueezing transform
- H: hop size (number of samples between each successive column of the synchrosqueezing transform)

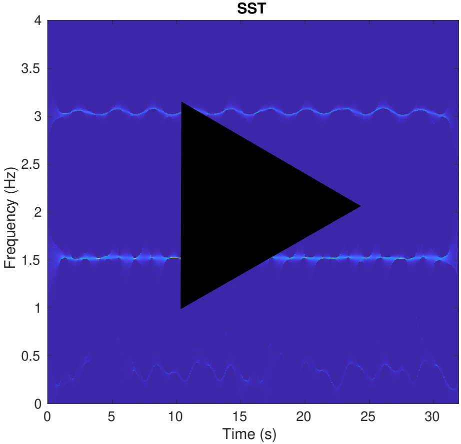
A general rule to determine the acceptable values of H for real-time implementation of update the boundary-free TF representation is

$$t_{\mathrm{forecast}} + \left\lceil \frac{L}{H} \right\rceil t_{\mathrm{SST}} < \frac{H}{f_s} \,.$$

Applications to Real Physiological Signals

PPG signal : real-time implementation

In this example, taking $H \geq 8$ samples is sufficient to ensure the feasibility of real-time implementation. It thus allows a maximum overlap of 98.4% of the window length.



Conclusion

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We propose an algorithm for the real-time reduction of boundary effects in time-frequency representations. We have shown that:

- the dynamic model is theoretically sufficient to extend AHM signals.
- the low running time allows real-time implementation;
- the algorithm is robust to noise;
- it can be applied to many time-frequency representations.

Perspectives

- Add a preliminary step that detect signal activity, and disable the forecasting step when necessary;
- accelerate the algorithm by optimizing the forecasting step;
- extend this strategy to more challenging biomedical signals, such as electroencephalogram (EEG).

Questions?